RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2017

THIRD YEAR [BATCH 2014-17]

Date : 08/05/2017 **MATHEMATICS (Honours)**

Time : 11 am – 3 pm **Paper :** VIII Full Marks : 70

[Use a separate Answer Book for <u>each group</u>]

<u>Group – A</u>

Answer <u>any five</u> questions from <u>Question Nos. 1 to 8</u> : [5×6]

1. A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place? 6

2. Two uniform rods AB and CD each of weight W and length a, are smoothly jointed at 'O' where OB = OD = b. The rods rest in a vertical plane with the ends A and C on a smooth table and the ends B and D are connected by a light string. Show that the reaction at the joint is $\frac{aW\tan\alpha}{2b}$, where 'α' is inclination of either rod to the vertical. 6

3. A telegraph wire is made of a given material, and such a length 'l' is stretched between two posts, distant 'd' apart and of same height, as will produce the least possible tension at the posts. Show that $l=\frac{d}{\lambda}\sinh\lambda$, where λ is given by the equation $\lambda\tanh\lambda=1$. 6

4. Two uniform straight rods PQ, $P'Q$, in all respects alike, are smoothly jointed at Q and P, P' carry small rings which slide on a smooth fixed parabolic wire whose axis is vertical and vertex upwards. Prove that in the symmetrical position of equilibrium, the angle either rod makes with the horizontal is $\sin^{-1}\left\{\frac{aW}{l(W+w)}\right\}$, where W is the weight of either rod, w of either ring, l the length of either rod and $4a$ the latus rectum of the parabola. 6

5. If a system of coplanar forces acting on a rigid body be in equilibrium and if the body be given any arbitrary infinitesimal virtual displacement in the plane of the forces then prove that the algebraic sum of the virtual works done by the forces is zero and conversely. 6

6. Forces X, Y, Z act along the three lines given by the equations $y = 0$, $z = c$; $z = 0$, $x = a$; $x = 0$, $y = b$; prove that the pitch of the equivalent wrench is $\frac{(aYZ+bZX+cXY)}{(X^2+Y^2+Z^2)}$. If the wrench reduces to a single force, show that the line of action of the force must lie on the hyperboloid $(x-a)(y-b)(z-c)-xyz=0$. 6

7. A sphere of weight W and radius r lies within a fixed spherical shell of radius R and a particle of weight w is attached to its highest point. Show that the equilibrium is stable if $W\geq\frac{R-2r}{r}w$. 6

8. Show that any system of forces acting on a rigid body can be reduced to a single force and a couple whose axis lies along the line of action of the force. 6

Answer <u>any two</u> questions from <u>Question Nos. 9 to 11</u> : [2×10]

9. a) Can a finite Boolean algebra $\{B, +, \bullet, ' \}$ have odd number of elements? Justify your answer. 2

   b) For any Boolean algebra, prove that $x = 0 \Leftrightarrow y = y' \cdot x + y \cdot x'$. 3

   c) A committee of three persons decides proposals by a majority of votes. One member has a voting weight 1 and others have weights 2, 2 respectively. Design a simple switching circuit, having minimum number of switches, so that light will glow when a majority of votes is cast in favour of the proposal. 3

   d) Express the Boolean expression $X + (X'Y' + X'Z)'$ in disjunctive normal form involving three variables *X, Y, Z.* 2

10. a) A C program contains the following declaration: 3

   static int *x*[] = {10, 20, 30, 40, 50, 60, 70, 80};

   (i) What is the meaning of *x*?

   (ii) What is the value of $(*x + 2)$?

   (iii) What is the value of $*(x + 5)$?

   b) A C program is given below: 2

```
# include <stdio.h>
void funct (int *p)
    {
        int i, sum = 0;
        for (i = 3; i<5; ++i)
                sum+= *(p+i);
        printf(" sum = %d \n",sum);
    }

main()
    {
        static int a[] = {10, 20, 30, 40, 50};
        funct (a);
    }
```

   (i) What kind of argument passed to funct?

   (ii) What value is displayed by printf statement within funct?

   c) Write a C program to store ASCII value of each characters of "Vidyamandira" in a data file test.dat. 5

11. a) Write a suitable program in C to find the sum of the infinite series: $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

   Correct to 4 decimal places for a given value of *x* using the concept of convergence of a sequence of partial sums. 5

   b) The following assignment statement, *a*, *b* and *c* are to be integer values. The conditional expression is given below:

   $c+ = (a > 0 \;\&\&\; a <= 10)? ++a : a / b;$

   Determine the value of *c* if

   (i) *a* = 1, *b* = 2, *c* = 3,

   (ii) *a* = 50, *b* = 10, *c* = 20. 2

c) What is the output of the following C-program? 3

```
#include<stdio.h>
#include<math.h>
main()
    {
        int a = 2, b = –7, c = 3, i;
        float d, r1, r2;
        d=b*b–4*a*c;
        if (d<0)
          i= –1;
        else if (d==0)
          i=0;
        else
          {
            i=1;
            d=sqrt(d);
          }
switch(i)
    {
        case –1:
            printf("Roots are imaginary\n");
            break;
        case 0:
            printf("Roots are equal \n");
            break;
        case 1:
        {
            printf("r1=%f\n",(–b+d) / (2*a));
                printf("r2=%f\n",(–b–d)/(2*a));
        }
    }
}
```

Group – B

[Attempt either Unit – I or Unit – II]

Unit - I

Answer any two questions from Question Nos. 12 to 14 : [2×10]

12. a) Let F, G be two subsets of $\mathbb{R}$ such that F is measurable and $m^*(F\Delta G) = 0$. Show that G is also measurable. [3]

b) If f be a measurable function on a measurable set $E(\subset \mathbb{R})$ and $f(x) \neq 0$ on E, prove that $\frac{1}{f}$ is also measurable on E. [3]

c) Let $E = [0,1]$ and $f : E \to \mathbb{R}$ be defined by

f(x) = 0, if x is rational

= n, if x is irrational and n is the total number of zeros immediately after the decimal point in the decimal expansion of x.

Show that f is measurable. [4]

13. a) Suppose $E \subseteq \mathbb{R}$. Show that the following statements are equivalent.

i) E is measurable i.e. $m^*(A) \geq m^*(A \cap E) + m^*(A \cap E')$, for each $A \subseteq \mathbb{R}$.

ii) $\forall \in > 0$, $\exists$ an open set $U \supseteq E$ such that $m^*(U - E) \leq \in$.

iii) $\exists$ a G_δ set $G \supseteq E$ such that $m^*(G - E) = 0$. [5]

b) Show that there is a non-measurable set in $\mathbb{R}$. [5]

14. Suppose C denotes the Cantor set contained in [0,1]. Prove that

a) C is uncountable. [3]

b) C is of measure zero. [2]

c) Each point of C is a point of condensation of C. [5]

OR,

Unit - II

Answer <u>any four</u> questions from <u>Question Nos. 15 to 20</u> : [4×5]

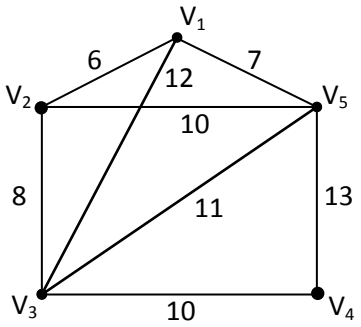
15. a) Prove that the number of odd degree vertices of a graph is always even.

Also prove that the number of edges of a complete graph with *n* vertices is $\frac{n(n-1)}{2}$. 2+1

b) Does there exist a tree *G* with 12 vertices such that the total degree of *G* is 24? Justify your answer. 2

16. a) Distinguish between walk and circuit in a graph. 2

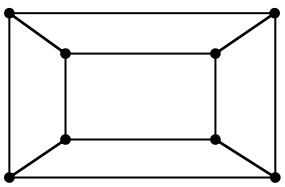
b) Find a minimal spanning tree of the following connected weighted graph *G* by applying Kruskal's algorithm: 3

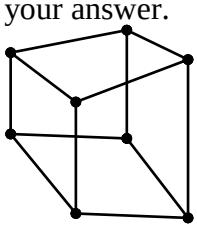


17. Prove that a connected planar graph with *n* vertices and *e* edges has $e - n + 2$ regions. 5

18. a) Prove that a tree with *n* vertices has $n - 1$ edges. 3

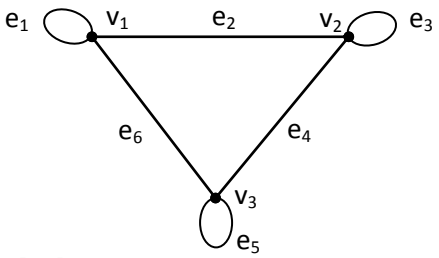
b) Are the following two graphs isomorphic? Justify your answer. 2





19. a) State and prove the necessary and sufficient condition for a graph to be an Euler graph. 4

b) Find an Euler circuit, if it exists in the following graph: 1



20. a) Define a bipartite graph and a binary tree. 1+1

b) Show that a bipartite graph does not contain any odd cycle. 3

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